

Probability Models (Ch.16):

- Example: (Insurance world)

Policyholder Outcome	Payout (\$)	Probability $P(X=x)$
Death	10,000	$\frac{1}{1000}$
Disability	5,000	$\frac{2}{1000}$
Neither	\emptyset	$\frac{997}{1000}$

- Random variable \rightarrow numeric value based on a random event
 \rightarrow symbolized by a capital letter
- Expected Value (Center, Mean)

$$\mu = E(X) = \sum x P(x)$$

From Example:

$$\begin{aligned} \mu &= (\$10,000)\left(\frac{1}{1000}\right) + (\$5,000)\left(\frac{2}{1000}\right) + (\emptyset)\left(\frac{997}{1000}\right) \\ &= \$20 \end{aligned}$$

- Spread (Standard Deviation and Variance)

- Variance $\rightarrow \sigma^2$

- Standard Deviation $\rightarrow \sigma$

- Variance Equation $\rightarrow \sigma^2 = \sum (x - \mu)^2 P(x)$

- St. Dev. Equation $\rightarrow \sigma = \sqrt{\sum (x - \mu)^2 P(x)}$

- From example:

$$\begin{aligned}\sigma^2 &= (10000 - 20)^2 \left(\frac{1}{1000}\right) + (5000 - 20)^2 \left(\frac{2}{1000}\right) + \\ &\quad (0 - 20)^2 \left(\frac{997}{1000}\right) \\ &= \$149,600\end{aligned}$$

$$\sigma = \sqrt{\text{Variance}} = \sqrt{\$149,600} = \$386.78$$

Worksheet 16-9:

Outcome	Spin \$5	Spin \$20	Spin \$50	Spin \$80
X	$\$20 - \$5 = \$15$	$\$20 - \$20 = \$0$	$\$20 - \$50 =$ $\$-30$	$\$20 - \$80 =$ $\$-60$
$P(X)$	0.5	0.25	0.125	0.125

$$\begin{aligned}
 c) \mu = E(X) &= (\$15)(0.5) + (\$0)(0.25) + \\
 &\quad (\$-30)(0.125) + (\$-60)(0.125) \\
 &= \$-3.75
 \end{aligned}$$

d) In the long run, the LLN suggests that the charity can expect to lose \$3.75 per spin.

$$\begin{aligned}
 e) \mu = E(X) &= (0.5)(\$25 - \$5) + \\
 &\quad (0.25)(\$25 - \$20) + \\
 &\quad (0.125)(\$25 - \$50) + \\
 &\quad (0.125)(\$25 - \$80) \\
 &= \$1.25
 \end{aligned}$$